Improved Simulated Annealing Algorithm for PAPR Reduction in OFDM Systems

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Abstract—The search for more computationally efficient algorithm is mandatory for PAPR reduction especially in OFDM systems employing a large number of sub-channels. Even though the proposed genetic algorithm in [2] has reduced the computational complexity of the search to obtain a sub-optimum solution but we found that there are other techniques such as the simulated annealing could also produce promising results and possibly with lower computational complexity. This paper has investigated the possibility of adopting an improved version of simulated annealing to achieve a more efficient PAPR reduction in OFDM systems.

I. INTRODUCTION

The most important problem associated with OFDM based modulation schemes is the inherent high peak-to-average power ratio (PAPR), which requires the use of highly linear power amplifier. The power limitation of the amplifier will cause the out-of-band noise and degradation on the OFDM signal. Today there exist many PAPR reductions schemes and in this paper a further investigation will focus on the use of the Peak Reduction Carriers (PRCs) first proposed by Lawrey [1]. The number of PRCs generated will be joining the data sub-channels to form the final OFDM signal as shown in Fig. 1.

![Fig. 1. Inserting PRCs into the OFDM sub-channels](image)

PRCs of different amplitude and phase will be generated such that a lower PAPR can be produced at the transmitter output. The amplitude \( Z \) and phase \( \phi \) of each sub-channel \( P_{\text{m}} \) are defined in equation (1), which can be represented in complex form.

\[
Ze^{j\phi} = I + jQ
\]  

(1)

The \( I \) and \( Q \) components are the parameters that determine the different PRCs generated for PAPR reduction. The searching for the best combinations of amplitude and phase for the set of PRCs is the main focus in this paper. The more sub-channels used in an OFDM system (i.e., 64 sub-channels and higher), the more PRC is required. Owing to that, a more efficient search strategy is required to obtain the desired sub-optimum PRCs. A genetic algorithm (GA) has been proposed by C.E.Tan and I.J.Wassell [2] as a search technique to quickly find the sub-optimum amplitude and phase of the PRCs configuration. The scheme allows the OFDM system to find a sub-optimum solution for PAPR reduction in a relatively efficient way.

An improved version of simulated annealing (SA) algorithm is proposed here as an alternative search technique to effectively find a sub-optimum solution for a lower PAPR. This is to further reduce the computational complexity of the search. Since simulated annealing is a metaheuristic search algorithm, a lot of choices are investigated to turn it into an appropriate algorithm suitable for the purpose of PAPR reduction. The simulated annealing control parameters can be fine-tuned into the best possible precision along the process of investigation because the precision of the values applied in the parameters setting have a significant effect upon the quality of the outcome. Most optimization algorithms use the two searching techniques: exploration to investigate new and unknown areas in the search space, and exploitation by makes use of knowledge found at previously visited points to help in finding a better point. The two requirements are contradictory, and a good search algorithm must find a trade-off between the two techniques.

II. THE SIMULATED ANNEALING SEARCHING METHOD

Simulated annealing is an optimization algorithm introduced in 1983 by Kirkpatrick et. al. [3]. In the OFDM system adopting SA, the state refer to the PRCs configuration and a PRCs state is denoted by the symbol \( \omega \). The cost function for the OFDM signal (combination of PRCs state and data carriers) is defined in equation (2), the \( \text{Max}(S) \) is the
maximum amplitude of the OFDM signal and the denominator of the equation $Mean(S)$ is the mean amplitude of the OFDM signal and the PAPR is measured in dB. Our goal is to search for a PRCs state that has minimum cost.

$$PAPR_{dB} = 20 \log_{10} \left( \frac{Max(S)}{Mean(S)} \right)$$

In the SA processes, the PRCs state $\omega$ is randomly permuted to obtain a new PRCs state $\omega'$, the probability of accepting $\omega'$ is determined by the change in cost and temperature $T$. The lower PAPR will always be accepted while higher PAPR will be rejected. If $\omega'$ is accepted, it becomes the current PRCs state but if it is not accepted, the current PRCs state $\omega$ will remain. Permutations are carried out until the system reaches an equilibrium distribution of PRCs state. As the search for a better sub-optimum PRCs state proceeds, $T$ will be decreasing and the sequence of $T$ dropping is referred to an annealing schedule. Lastly, the stopping criterion is to stop the algorithm when $T$ or PAPR drops below the predefined threshold.

Simulated annealing requires a problem-specific mean of generating a new PRCs solution from an old PRCs solution through permutation. Evaluating a new PRCs solutions that are near an existing PRCs solution may be more efficient, which may give a performance advantages to simulated annealing, when compared to genetic algorithm, if evaluating recombined (crossover and mutation) PRCs solutions, it is not as efficient. Simulated annealing seems to be able to obtain good PRCs solutions in a short time period, but not be able to achieve a great improvement if given more time. In practical term, all algorithms will produce a better PRCs solution if given more execution time, unfortunately time is always a critical issue in a real time system. Owing to that, execution time should be the key element for algorithm performance determination and the selection of optimum parameters for efficient simulated annealing implementation is mandatory.

III. THE IMPROVED AND OPTIMISED PARAMETERS FOR SIMULATED ANNEALING

The elements of simulated annealing that can be further developed and optimised are the population size, the move class, the annealing schedule, the acceptance criterion and the equilibrium distribution. The functional blocks of an improved version of simulated annealing are shown in Fig. 2. An initial random population of $N$ samples of PRCs state space will be first generated, and each sampled PRCs states called the random walkers of length $m$ will perform the neighbouring process through randomly change of carriers position within the same PRCs state space to generate a new PRCs state. The acceptance rule will be used to evaluate the probability of accepting the new PRCs state if only if a small uphill move in the PAPR is detected, and those accepted PRCs states will undergo a mutation process. For the new PRCs state with a downhill move or large uphill move in PAPR, mutation will not be introduced. Any new PRCs state generated will be compared to the best-so-far PRCs state (obtained previously). After each iterations, the equilibrium of the new PRC states will determine whether the temperature should remain unchanged or drop to a lower temperature. The SA will continue the next iteration with another population of random walkers if further PAPR reduction is required, and the process of search will be carried out until the targeted PAPR is achieved. Upon completion of all the required iterations the sub-optimum PRCs obtained will be inserted into the OFDM transmitter together with the data symbols.

![Fig. 2. Simulated annealing search for sub-optimum PRCs](image-url)

By introducing the idea of population, simulated annealing randomly samples the PRCs state space to initially generate members of the population, and called the random walkers. Adaptation of the population into simulated annealing means sharing the search among several random walkers instead of just one walker for all the searches and picking the best PAPR result obtained at the end of the search. The information obtained from several identical random walkers ideally performed simultaneously and the collective information extracted from a population is useful to further improve the search efficiency of simulated annealing. Simulated annealing works by simulating a number of random walkers on a set of PRCs state space, it searches the sampled PRCs state space looking for the best-so-far PRCs state. Fig. 3 shows the average PAPR reduction performance with various walker sizes. The results show that increasing the number of walker will increase the search performance but the increment is not in a linear pattern. At some points, a larger number of walkers perform worse than smaller number of walkers, for example the population size of 3 random walkers has the best performance.
The move class is a procedure of generating a neighbour PRCs state for a given PRCs state, the carrier position permutation method and carrier mutation method are both the move class method introduced for the simulated annealing. Introducing mutation into simulated annealing will achieve a better PAPR reduction as shown in Fig. 4, but no further PAPR reduction can be achieved if introducing mutation after an average of 537 iterations.

At each instant during the simulation, we have the current PRCs state from which we select a random neighbour PRCs state, the acceptance rule determines whether to accept the move to neighbour PRCs state and proceed from there or reject the move and stay at the current PRCs state.

\[ \Delta PAPR = PAPR (S') - PAPR (S) \]  \hspace{1cm} (3)

The equation (3) defines the uphill and downhill moves respectively, according to the conditions \( \Delta PAPR > 0 \) or \( \Delta PAPR \leq 0 \). The symbol \( S \) defines the current PRCs state with data carriers and the symbol \( S' \) defines the new PRCs state with data carriers. The downhill moves will always be accepted while small uphill moves will be accepted with a mutation, this is important to avoid getting trapped in a local minimum. The annealing schedule for the simulated annealing is the rate of decreasing temperature \( T \), and the acceptance probability is reversely proportional to the rate of temperature. The chance of accepting any uphill move of PRCs state is higher when \( T \) is at higher stage but it decreases as \( T \) decrease. The exponential annealing schedule method is the choice of decreasing temperature, and carefully optimized annealing schedule can make a significant difference on the PAPR reduction.

To distribute a set of random walkers in the best possible way through the control of all the elements of SA for a given fixed amount of computing resources will be the main objective of our research, investigating the optimal values of all the parameters for the important elements of SA such as the population sizes, the move class, the annealing schedule, the acceptance probability and the equilibrium distribution are mandatory in order to formulate a more efficient simulated annealing algorithm which can be used to quickly compute the best possible PRCs for the lowest possible PAPR within an allowed computational complexity. In particular, it is better to use acceptance rules for which a good annealing schedule is known and varying the acceptance rule will also vary the resulting equilibrium distribution. Different annealing schedule for either exponentially or sequentially can both alter the way that the acceptance probability accepting any uphill move of PRCs states along the process of search for better PAPR reduction. The exponential method of annealing has been selected to form the exponential of acceptance rule and eventually the equilibrium will distribute exponentially.

A. The Annealing Schedule

The annealing schedule or cooling is to determine the rate of temperature reduction (in discrete steps) from an initially high temperature to a lower temperature and it is computed through the equation (4). The parameter \( T_t \) is the new temperature, while \( T_{t-1} \) is the current temperature and \( P \) determines the overall rate of cooling.

\[ T_t = \frac{1}{1 + P \cdot \frac{T_{t-1}}{T_t}} \]  \hspace{1cm} (4)

The rate of cooling must match the natural time scales of the PAPR reduction problem. A rapid cooling tends to narrow the PRCs search space to only the limited sample of PRCs state space because it lower the probability for mutation, and consequently reduces the chance of generating a new
PRCs search space. Whereas, slower cooling rate allows more mutation and eventually more PRCs state space will be explored, and this will prevent the search from being trapped in a local minimum. Fig. 5 shows the PAPR performance of two different rates of annealing schedule indicated by the two different setting of parameter $P$. The setting of $P = 0.05$ indicates a lower rate of annealing schedule, while $P = 0.9$ indicates a high cooling rate. The rate of mutation changes reverse proportionally to the rate of annealing schedule as indicated in Fig. 6. Therefore, the slower cooling rate with an exponential increase of mutation rate achieves better sub-optimum results for PAPR reduction. For whatever schedule used, the start and stop criteria have to be set according to the natural scales of the PAPR reduction problem. The starting temperature is set to 0.9 as a suitable starting rate of cooling and the stopping temperature is set by specifying the number of temperature steps for stop, or if no further PAPR reduction is achieved in the last few step specified, and or the targeted PAPR has been achieved.

Fig. 5. The average PAPR achieved with different rate of annealing schedule

Fig. 6. The average rate of mutation using different rate of annealing schedule

B. The Acceptance Probability

The acceptance rule (also known as Tsallis acceptance probability) suggested by Penna [5], Tsallis and Stariolo [6] and [7] is adapted in simulated annealing for the PRCs search scheme to effectively accept the next PRCs state. By introducing the Tsallis acceptance rule, it allow more effective way of introducing mutation, so that the adaptation of mutation in the simulated annealing will not rule out the purpose of reducing PAPR as well as the computational complexity.

\[
p_r(A_{\text{PAPR}}) = \begin{cases} 
1 & \text{if } A_{\text{PAPR}} \leq 0 \\
\left(1 - (1-q)\cdot \frac{A_{\text{PAPR}}}{T}\right)^{\frac{1}{q}} & \text{if } 0 < A_{\text{PAPR}} < 1 \text{ and } 0 < 1-q < 1 \\
0 & \text{if } A_{\text{PAPR}} > 1 \text{ and } 0 < 1-q \leq 1 
\end{cases}
\]  

(5)

The parameter $q$ is a problem dependent positive constant required by the acceptance rule and the randomly generated number will be $0 < r < 1$, only if the equation (6) is true, then mutation will take place in which it randomly changes the PRCs state. The rule of accepting the mutation will occasionally allow large moves in PAPR changes. The small value of $q = 0.1$ has been selected from $0 < q < 1$ for optimal value because probability of accepting a small uphill on the PAPR become less as $q$ decreases in which less mutation will be introduced. This is important because high rate of mutation will increase the computational complexity. As mutation rate is proportional to acceptance probability, the rate of mutation will eventually keep as lower as possible. Thus, the parameter $q$ is the parameter for setting the mutation rate. The probability of accepting mutation can be greatly increased if $q$ is set to a higher value. However, the quality of PAPR results is roughly independent of $q$ as shown in Fig. 7 but the rate of mutation increase proportionally to the parameter $q$ as shown in Fig. 8.

Fig. 7. PAPR performance are roughly independent of $q$
Fig. 8. Mutation rate increase proportionally to parameter $q$

Therefore, the lower value of $q$ is the best selection for the acceptance probability because speed improves with the decreasing of $q$ as less number of mutation introduced reduce the computational complexity.

C. The Equilibrium Distribution

The number of permutation attempts at each temperature is defined in equation (7).

$$\Delta \text{Mean}_{\text{papr}} < \frac{C \times \delta}{\sqrt{W}}$$  \hspace{1cm} (7)

The collective information of the best-so-far PAPR after each temperature is computed to calculate the mean and called best-so-far mean PAPR. The change of the best-so-far mean PAPR between the best-so-far mean PAPR of the previous temperature to the mean PAPR computed from each population of the current temperature is defined by $\Delta \text{Mean}_{\text{papr}}$ and if equation (7) is true, then simulated annealing will run equilibrium steps at a fixed temperature until it found a lower PAPR otherwise it will run the maximum equilibrium steps.

The fraction of mean PAPR after the simulated annealing has equilibrated at previous temperature will be set as a target for equilibration for the next temperature step. The symbol $\delta$ define the standard deviation, the symbol $W$ refers to the number of random walkers and the parameter $C$ is the problem dependent positive constant used to control the equilibration at each temperature steps. The parameter value for $C$ equal to 1.0 with less execution time is selected as an optimum setting for the algorithm as shown in Fig. 9. At any fixed temperature, the probabilities the simulated annealing will be in each of its possible new PRCs states at the next instant given its current PRCs state is stationary but when the temperature is decreased, the probabilities change. The probabilities of accepting new PRCs states will be higher when temperature is high, while the probabilities decrease when temperature decreases. As a result, if temperature is lowered, the mean PAPR decreases because less probabilities of accepting any high uphill move and only the PRCs states with probabilities near one is accepted, and it is also less likely to obtain any lower PAPR as more PAPR reduction are required when temperature is at a lower stage. Eventually, simulated annealing will almost run at maximum equilibrium steps at lower temperature stages to search for a lower PAPR.

IV. COMPUTATIONAL COMPLEXITY

After investigating the optimum parameters for the improved simulated annealing base on the number of iterations for the OFDM system adopting 16 PRCs using 256-QAM scheme in the 64 sub-channel with 48 data carriers using QPSK. The computational complexity for the system using IFFT size of 128 was further evaluated according to the execution time required for the PAPR reduction as shown in Fig. 10.

Fig. 9. The average PAPR achieved from different setting of parameter $C$

Fig. 10. PAPR reduction as a function of the execution time

The improved simulated annealing algorithm using a CPU having performance of 318 MIPS Dhrystone and 333 MFLOPS Whetstone can achieve an average PAPR of 4.23dB with only less then 2 seconds, but it
will require more execution time for more PAPR reduction. Obviously, the more iterations undertaken mean more execution time will be required which also mean that more reduction of PAPR. Although more PAPR reduction can be obtained, the requirement for computational complexity have to be increased.

V. CONCLUSIONS

The improved simulated annealing has been proven as another effective algorithm suitable for PAPR reduction using PRCs scheme. The PAPR reduction in OFDM system employing a large number of sub-channels have been investigated through reducing the computational complexity by mean of the number of iterations and execution time. The important elements of SA such as the population size, the move class, the annealing schedule, the acceptance rule and the equilibrium distribution are tested for the optimum parameters suitable for the PRCs scheme to obtain a more computational efficient algorithm.

REFERENCES


